

A Versatile Characterization of Causal Relationships

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Abstract. This paper introduces a framework for representing versatile temporal relationships between events and their effects. The framework is based on a simple time model which characterizes each time element as a subset of the set of real numbers and allows expression of both absolute time values and relative temporal relations. The formalism presented here formally specifies the so-called most general temporal constraint (GTC), which guarantees the common-sense assertion that “the beginning of the effect cannot precede the beginning of the cause”. It is shown that there are in fact 8 possible causal relationships which satisfy GTC, including cases where, on the one hand, effects start simultaneously with, during, immediately after, or some time after their causes, and on the other hand, events end before, simultaneously with, or after their causes. The causal relationships characterized in this paper are versatile enough to subsume those representatives in the literature.

1 Introduction

Representing and reasoning about events and their effects is essential in modeling the dynamic aspects of the world. Over the past 40 decades, a multitude of alternative formalisms have been proposed in this area, including McCarthy and Hayes’ framework of the situation calculus [17, 18], McDermott’s temporal logic [19], Allen’s interval based theory [1, 2], Kowalski and Sergot’s event calculus [11], Shoham’s point-based reified logic and theory [26, 27], and Terenziani and Torasso’s theory of causation [28]. In particular, noticing that temporal reasoning plays an important role in reasoning about actions/events and change, a series of revised formalisms have been introduced to characterize richer temporal features in the situation calculus or the event calculus, such that of Lifschitz [12], of Sandewall [23], of Schubert [24], of Gelfond et al. [10], of Lin and Shoham [13], of Pinto and Reither [21, 22], of Miller and Shanahan [20, 25], and of Baral et al. [5, 6, 7].

In most existing formalisms for representing causal relationships between events and their effects, such as the situation calculus and the event calculus, the result of an event is represented by the effect takes place immediately after the occurrence of the event. However, as noted by Allen and Ferguson [3], temporal relationships between events and their effects can in fact be quite complicated. In some cases, the effects of an event take place immediately after the end of the event and remain true until some further events occur. E.g., in the block-world, as soon as the action “moving a block from the top of another block onto the table” is completed, the block being moved

should be on the table (immediately). However, sometimes there may be a time delay between an event and its effect(s). E.g., 30 seconds after you press the button at the crosswalk, the pedestrian light turns to green [10]. Also, in some other cases, the effects of an event might start to hold while the event is in progress, and stop holding before or after the end of the event. Examples can be found later in the paper.

The objective of this paper is to propose a framework, which allows expression of versatile temporal causal relationships between events and their effects. As the temporal basis for the formalism, a simple point-based time model is presented in section 2, allowing expression of both absolute time values and relative temporal relations. In section 3, fluents and states are associated with times in the manner of temporal reification [16, 26]. Section 4 deals with representation of event/action and change, as well as temporal constraints on the causal relationships between events and their effects. Finally, section 5 concludes the paper.

2 A Simple Time Model

In what follows in this paper, we shall use $\mathbf{R}$ to denote the set of real numbers, and $\mathbf{T}$, the set of time elements. Each time element t is defined as a subset of $\mathbf{R}$ and must be in one of the following four forms:

$$\begin{aligned} (p_1, p_2) &= \{p \mid p \in \mathbf{R} \wedge p_1 < p < p_2\} \\ [p_1, p_2) &= \{p \mid p \in \mathbf{R} \wedge p_1 \leq p < p_2\} \\ (p_1, p_2] &= \{p \mid p \in \mathbf{R} \wedge p_1 < p \leq p_2\} \\ [p_1, p_2] &= \{p \mid p \in \mathbf{R} \wedge p_1 \leq p \leq p_2\} \end{aligned}$$

In the above, p_1 and p_2 are real numbers, and we shall call them the left-bound and right-bound of time element t , respectively. The absolute values as for the left and/or right bounds of some time elements might be unknown. In this case, real number variables are used for expressing relative relations to other time elements.

In this paper, if the left-bound and right-bound of time element t are the same, we shall call t a time point, otherwise t is called a time interval. Without confusion, we shall take time element $[p, p]$ as identical to p . Also, if a time element is not specified as open or closed at its left (right) bound, we shall use “<” instead of “(” and “[” as for its left bracket; similarly, we shall use “>” instead of “)” and “]” as for its right bracket. In addition, we define the duration of a time element t , $\text{Dur}(t)$, as the distance between its left bound and right bound. In other words:

$$(2.1) \ t = \langle p_1, p_2 \rangle \Rightarrow \text{Dur}(t) = p_2 - p_1$$

Following Allen’s terminology [1], we shall use Meets to denote the immediate predecessor order relation over time elements:

$$(2.2) \ \text{Meets}(t_1, t_2) \Leftrightarrow \exists p_1, p_2 \in \mathbf{R} (t_1 = \langle p_1, p \rangle \wedge t_2 = [p, p_2 \rangle \vee t_1 = \langle p_1, p \rangle \wedge t_2 = (p, p_2 \rangle)$$

It is easy to see that the intuitive meaning of $\text{Meets}(t_1, t_2)$ is that, on the one hand, time elements t_1 and t_2 don’t overlap each other (i.e., they don’t have any part in common, not even a point); on the other hand, there is not any other time element standing between them.

N.B. For any two time elements t_1 and t_2 such that $\text{Meets}(t_1, t_2)$, t_1 and t_2 define a unique time element as the “ordered-union” of t_1 and t_2 , denoted as $t_1 \oplus t_2$.

Analogous to the 13 exclusive relations introduced by Allen for intervals [1, 2], in this paper, we shall use **TR** to denote the set of exclusive temporal order relations over time elements including both time points and time intervals:

$$\mathbf{TR} = \{\text{Equal, Before, After, Meets, Overlaps, Overlapped-by, Met-by, Starts, Started-by, During, Contains, Finishes, Finished-by}\}$$

It is important to note that, for a given pair of time elements, some of the relations in **TR** may be non-applicable. In fact, when the pair of time elements, t_1 and t_2 , are specified as a point and a point, a point and an interval, an interval and a point, and an interval and an interval, respectively, all the exclusive temporal order relations between t_1 and t_2 can be classified into the following four groups, which we shall call the Comprehensive Temporal Order Relations (CTOR):

- 3 relations relating a point to a point:
{Equal, Before, After}
- 7 relations relating a point to an interval:
{Before, After, Meets, Met-by, Starts, During, Finishes}
- 7 relations relating an interval to a point:
{Before, After, Meets, Met-by, Started-by, Contains, Finished-by}
- 13 relations relating an interval to an interval:
{Equal, Before, After, Meets, Met-by, Overlaps, Overlapped-by, Starts, Started-by, During, Contains, Finishes, Finished-by}

The definition of the derived temporal order relations in terms of the single relation *Meets* is straightforward [4]. In fact:

$$\begin{aligned} \text{Equal}(t_1, t_2) &\Leftrightarrow \exists t', t'' \in \mathbf{T} (\text{Meets}(t', t_1) \wedge \text{Meets}(t', t_2) \wedge \text{Meets}(t_1, t'') \wedge \text{Meets}(t_2, t'')) \\ \text{Before}(t_1, t_2) &\Leftrightarrow \exists t \in \mathbf{T} (\text{Meets}(t_1, t) \wedge \text{Meets}(t, t_2)) \\ \text{Overlaps}(t_1, t_2) &\Leftrightarrow \exists t, t_3, t_4 \in \mathbf{T} (t_1 = t_3 \oplus t \wedge t_2 = t \oplus t_4) \\ \text{Starts}(t_1, t_2) &\Leftrightarrow \exists t \in \mathbf{T} (t_2 = t_1 \oplus t) \\ \text{During}(t_1, t_2) &\Leftrightarrow \exists t_3, t_4 \in \mathbf{T} (t_2 = t_3 \oplus t_1 \oplus t_4) \\ \text{Finishes}(t_1, t_2) &\Leftrightarrow \exists t \in \mathbf{T} (t_2 = t \oplus t_1) \\ \text{After}(t_1, t_2) &\Leftrightarrow \text{Before}(t_2, t_1) \\ \text{Overlapped-by}(t_1, t_2) &\Leftrightarrow \text{Overlaps}(t_2, t_1) \\ \text{Started-by}(t_1, t_2) &\Leftrightarrow \text{Starts}(t_2, t_1) \\ \text{Contains}(t_1, t_2) &\Leftrightarrow \text{During}(t_2, t_1) \\ \text{Finished-by}(t_1, t_2) &\Leftrightarrow \text{Finishes}(t_2, t_1) \\ \text{Met-by}(t_1, t_2) &\Leftrightarrow \text{Meets}(t_2, t_1) \end{aligned}$$

For the convenience of expression, we define two non-exclusive temporal relations as below:

$$\begin{aligned} \text{In}(t_1, t_2) &\Leftrightarrow \text{Starts}(t_1, t_2) \vee \text{During}(t_1, t_2) \vee \text{Finishes}(t_1, t_2) \\ \text{Sub}(t_1, t_2) &\Leftrightarrow \text{Equal}(t_1, t_2) \vee \text{In}(t_1, t_2) \end{aligned}$$

Another important fact needs to be pointed out is that the distinction between the assertion that “point p Meets interval t ” and the assertion that “point p Starts interval t ” is

critical: while $\text{Starts}(p, t)$ states that point p is the starting part of interval t , $\text{Meets}(p, t)$ states that point p is one of the immediate predecessors of interval t but p is not a part of t at all. In other words, $\text{Starts}(p, t)$ implies interval t is left-closed at point p , and $\text{Meets}(p, t)$ implies interval t is left-open at point p . Similarly, this applies to the distinction between the assertion that “interval t is Finished-by point p ” and the assertion that “interval t is Met-by point p ”, i.e., $\text{Finished-by}(t, p)$ implies interval t is right-closed at point p , and $\text{Met-by}(t, p)$ implies interval t is right-open at point p .

As mentioned earlier, the simple point-based time model introduced here allows the openness (or closeness) of some interval at their left and/or right bounds to be unspecified. Such an approach provides a satisfactory representation of possibly incomplete relative temporal knowledge, and hence retains the appealing characteristics of interval-based [1], and point&interval-based [15] temporal systems. Specially, it can successfully bypass puzzles like the so-called Dividing Instant Problem [1, 8, 9, 14, 29].

3 Fluents and States

Representing the dynamic aspects of the world usually involves reasoning about various states of the world under consideration. In this paper, we shall define a state (denoted by, possibly scripted, s) of the world in the discourse as a collection of fluents (denoted by, possibly scripted, f), where a fluent is simply a Boolean valued proposition whose truth-value is dependent on the time.

The set of fluents, $\mathbf{F}$, is defined as the minimal set closed under the following two rules:

$$(3.1) f_1, f_2 \in \mathbf{F} \Rightarrow f_1 \vee f_2 \in \mathbf{F}$$

$$(3.2) f \in \mathbf{F} \Rightarrow \text{not}(f) \in \mathbf{F}$$

In order to associate a fluent with a time element, we shall use $\text{Holds}(f, t)$ to denote that fluent f holds true over time t .

As pointed out by Allen and Ferguson [3], as well as by Shoham [26], there are two ways we might interpret the negative sentence. In what follows, the sentence-negation will be symbolized “ $\neg$ ”, e.g., $\neg \text{Holds}(t, f)$, distinguished from the negation of fluents, e.g., $\text{not}(f)$ [9]. In the weak interpretation, $\neg \text{Holds}(t, f)$ is true if and only if it is not the case that f is true throughout t , and hence $\neg \text{Holds}(t, f)$ is true if f changes truth-values over time t . In the strong interpretation of negation, $\neg \text{Holds}(t, f)$ is true if and only if f holds false throughout t , so neither $\text{Holds}(t, f)$ nor $\neg \text{Holds}(t, f)$ would be true in the case that fluent f is true over some sub-interval of t and also false over some other sub-interval of t .

In this paper, we take the weak interpretation of negation as the basic construct:

$$(3.3) \text{Holds}(f, t) \Rightarrow \forall t' (\text{Sub}(t', t) \Rightarrow \text{Holds}(f, t'))$$

That is, if fluent f holds true over a time element t , then f holds true over any part of t .

$$(3.4) \text{Holds}(f_1 \vee f_2, t) \Leftrightarrow \text{Holds}(f_1, t) \vee \text{Holds}(f_2, t)$$

That is, if fluent f_1 or fluent f_2 holds true over time t , then at least one of them holds true over time t .

$$(3.5) \text{ Holds}(f, t_1) \wedge \text{Holds}(f, t_2) \wedge \text{Meets}(t_1, t_2) \Rightarrow \text{Holds}(f, t_1 \oplus t_2)$$

That is, if fluent f holds true over two time elements t_1 and t_2 that meets each other, then f holds over the ordered-union of t_1 and t_2 .

Following the approach proposed in [25], we use $\text{Belongs}(f, s)$ to denote that fluent f belongs to the collection of fluents representing state s :

$$(3.6) s_1 = s_2 \Leftrightarrow \forall f(\text{Belongs}(f, s_1) \Leftrightarrow \text{Belongs}(f, s_2))$$

That is, two states are equal if and only if they contain the same fluents.

$$(3.7) \exists s \forall f(\neg \text{Belongs}(f, s))$$

That is, there exists a state that is an empty set.

$$(3.8) \forall s_1 f_1 \exists s_2 (\forall f_2 (\text{Belongs}(f_2, s_2) \Leftrightarrow \text{Belongs}(f_2, s_1) \vee f_1 = f_2))$$

That is, any fluent can be added to an existing state to form a new state.

Without confusion, we also use $\text{Holds}(s, t)$ to denote that state s holds true over time t , provided:

$$(3.9) \text{ Holds}(s, t) \Leftrightarrow \forall f(\text{Belongs}(f, s) \Rightarrow \text{Holds}(f, t))$$

4 Events, Effects and Causal Relationships

The concepts of change and time are deeply related since changes are caused by events occurring over the time. In order to express the occurrence of events (denoted by e , possibly scripted), following Allen's approach [2], we use $\text{Occur}(e, t)$ to denote that event e occurs over time t , and impose the following axiom:

$$(4.1) \text{ Occur}(e, t) \Rightarrow \forall t'(\text{In}(t', t) \Rightarrow \neg \text{Occur}(e, t'))$$

We shall use formula $\text{Changes}(t_1, t, t_2, s_1, e, s_2)$ to denote a causal law, which intuitively states that, under the precondition that state s_1 hold over time t_1 , the occurrence of event e over time t will change the world from state s_1 into state s_2 , which holds over time t_2 . Formally, we impose the following axiom about causality to ensure that if the precondition of a causal law holds and the event happens, then the effect expected to be caused must appear:

$$(4.2) \text{ Changes}(t_1, t, t_2, s_1, e, s_2) \wedge \text{Holds}(s_1, t_1) \wedge \text{Occur}(e, t) \Rightarrow \text{Holds}(s_2, t_2)$$

In order to characterize temporal relationships between events and their effects, we impose the following temporal constraints:

$$(4.3) \text{ Changes}(t_1, t, t_2, s_1, e, s_2) \Rightarrow \text{Meets}(t_1, t) \wedge (\text{Meets}(t_1, t_2) \vee \text{Before}(t_1, t_2))$$

It is important to note that axiom (4.3) presented above actually specifies the so-called (most) general temporal constraint (GTC) (see [2, 19, 27, 28]). Such a GTC guarantees the common-sense assertion that “the beginning of the effect cannot precede the beginning of the cause”.

There are in fact 8 possible temporal order relations between times t_1 , t and t_2 which satisfy (4.3). These are illustrated in Figure 1 as below:

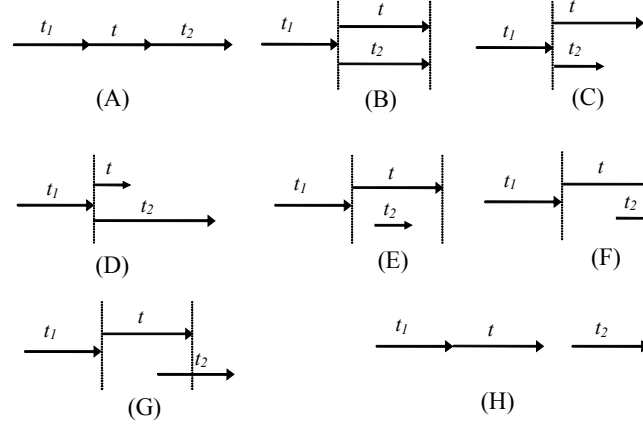


Figure 1. Temporal order relations between times t_1 , t and t_2 which satisfy (4.3).

- Case (A) where the effect becomes true immediately after the end of the event and remains true for some time after the event. E.g., the event of putting a book on the table has the effect that the book is on the table immediately after the event is completed.
- Case (B) where the effect holds only over the same time over which the event is in progress. E.g., as the effect of the event of pressing the horn of a car, the horn makes sounds only when the horn is being pressed.
- Case (C) where the beginning of the effect coincides with the beginning of the event, and the effect ends before the event completes. E.g., consider the case where a landmine is set on the first half of a bridge. If someone is walking through the bridge, he will be in danger just over the first half of the bridge crossing.
- Case (D) where the beginning of the effect coincides with the beginning of the event, and the effect remains true for some time after the event. E.g., as the effect of the event of pressing the button of the bell on the door (say, for one second), the bell sounds a tune for fifteen seconds.
- Case (E) where the effect only holds over some time during the progress of the event. E.g., he goes through a wall of tiredness over the fiftieth minute of the event of running for four hours.
- Case (F) where the effect becomes true during the progress of the event and remains true until the event completes. E.g., consider the event of discharging some water from a basket by means of lifting one side of the basket. In the case where the basket is not full, the effect that the water flows out takes place only after it has been lifted to the edge of the basket, and will keep flowing out until the event ends.
- Case (G) where the effect becomes true during the progress of the event and remains true for some time after the event. E.g., he becomes tired for days after the thirtieth minute of the event of running along the track for three hours.

- Case (H) where there is a time delay between the event and its effect. E.G., 25 seconds after the button at the crosswalk is pressed, the pedestrian light turns to yellow; and after another 5 seconds, it turns to green.

As mentioned in the introduction, various theories have been proposed for representing and reasoning about action/event and change. However, the temporal causal relationships between events and their effects as specified in most of the existing formalism are quite limited. An exception is the relatively general theory of Time, Action-Types, and Causation, introduced by Terenziani and Torasso's in the middle of last 90s [28]. Due to the limit to the length of the paper, in what follows, we shall briefly demonstrate that the causal relationships characterized in this paper are more general than that of Terenziani and Torasso, and therefore versatile enough to subsume those representatives in the literature. In fact:

If t and t_2 are specified as a point and a point, a point and an interval, an interval and a point, and an interval and an interval, respectively, by applying the CTOR as classified in section 2, we can reach the following four theorems straightforwardly:

$$\begin{aligned} \text{(Th1)} \quad & \text{Changes}(t_1, t, t_2, s_1, e, s_2) \wedge \text{Dur}(t) = 0 \wedge \text{Dur}(t_2) = 0 \\ & \Rightarrow \text{Equal}(t, t_2) \vee \text{Before}(t, t_2) \end{aligned}$$

That is, if the event and the effect are both punctual, then either the event precedes (strictly) the effect, or they coincide with each other (i.e., they happens simultaneously at the same time point).

$$\begin{aligned} \text{(Th2)} \quad & \text{Changes}(t_1, t, t_2, s_1, e, s_2) \wedge \text{Dur}(t) = 0 \wedge \text{Dur}(t_2) > 0 \\ & \Rightarrow \text{Starts}(t, t_2) \vee \text{Meets}(t, t_2) \vee \text{Before}(t, t_2) \end{aligned}$$

That is, if the event is punctual and the effect is durative, then either the event precedes (immediately or strictly) the effect, or the event coincides with the beginning of the effect.

$$\begin{aligned} \text{(Th3)} \quad & \text{Changes}(t_1, t, t_2, s_1, e, s_2) \wedge \text{Dur}(t) > 0 \wedge \text{Dur}(t_2) = 0 \\ & \Rightarrow \text{Started-by}(t, t_2) \vee \text{Contains}(t, t_2) \\ & \quad \vee \text{Finished-by}(t, t_2) \vee \text{Meets}(t, t_2) \vee \text{Before}(t, t_2) \end{aligned}$$

That is, if the event is durative and the effect is punctual, then either the event precedes (immediately or strictly) the effect, or the effect coincides with either the beginning or the end of the event, or the effect happens during the event's occurrence.

$$\begin{aligned} \text{(Th4)} \quad & \text{Changes}(t_1, t, t_2, s_1, e, s_2) \wedge \text{Dur}(t) > 0 \wedge \text{Dur}(t_2) > 0 \\ & \Rightarrow \text{Started-by}(t, t_2) \vee \text{Contains}(t, t_2) \vee \text{Finished-by}(t, t_2) \vee \text{Equal}(t, t_2) \\ & \quad \vee \text{Starts}(t, t_2) \vee \text{Overlaps}(t, t_2) \vee \text{Meets}(t, t_2) \vee \text{Before}(t, t_2) \end{aligned}$$

That is, if both the event and the effect are durative, then the beginning of the event either precedes (immediately or strictly) or coincides with the beginning of the effect, where the end of the event can either precedes (immediately or strictly), coincides with, or succeeds (immediately or strictly) the end of the effect.

It is easy to see that (Th1) and (Th4) are equivalent to Terenziani and Torasso's Theorem 4 (p.541, [28]) and Theorem 1 (p.540, [28]), respectively, while (Th2) and (Th3) can be seen as the extension to Terenziani and Torasso's Theorem 3 and

Theorem 2 (p.541, [28]), respectively. This is due to the fact that, while the “Meets” relation between a punctual event and a durative effect, and between a durative event and a punctual effect, is accommodated in (Th2) and (Th3), respectively, Terenziani and Torasso’s Theorem 3 and Theorem 2 do not allow such relations.

In fact, follow Terenziani and Torasso’s Theorem 3, either there must be a gap between the punctual cause and its durative effect, or the punctual cause must coincide with the beginning part of its durative effect. In other words, the interval over which the effect happens must be either “After” or “closed at” the point at which the cause happens. Therefore, the case where a punctual cause “Meets” its durative effect (that is, the interval over which the effect happens is “open” at the point at which the cause happens) is not allowable. However, consider the following example:

Immediately after the power was switched on, a robot that had been stationary started moving.

If we use $s_{\text{Stationary}}$ to represent the state that “the robot was stationary”, e_{SwitchOn} to represent the event that “the power was switched on”, and s_{Moving} to represent the corresponding effect that “the robot was moving”, then

$$\text{Changes}(t_{\text{Stationary}}, t_{\text{SwitchOn}}, t_{\text{Moving}}, s_{\text{Stationary}}, e_{\text{SwitchOn}}, s_{\text{Moving}})$$

should be consistent with:

$$\text{Meets}(t_{\text{SwitchOn}}, t_{\text{Moving}})$$

That is, the “Switching” point t_{SwitchOn} is immediately followed by the “Moving” interval, but not included in the “Moving” interval itself. In other word, the robot was moving immediately after the “Switching” point t_{SwitchOn} , but at the time point when the power was switching on, the robot was not moving. Obviously, such a scenario cannot be expressed in Terenziani and Torasso’s Theorem 3.

Similarly, in Terenziani and Torasso’s Theorem 2, the case where a durative event “Meets” its punctual effect (that is, the interval over which the cause happens is “open” at the point at which the effect happens) is not allowable. Then again, consider the following example:

Immediately after the ball being falling down from the air, it touched the ground.

If we use s_{InAir} to represent the precondition that “the ball was at a certain position in the air”, $e_{\text{FallingDown}}$ to represent the event that “the ball was falling down”, and $s_{\text{TouchGround}}$ to represent effect that “the ball touched the ground”, respectively, then

$$\text{Changes}(t_{\text{InAir}}, t_{\text{FallingDown}}, t_{\text{TouchGround}}, s_{\text{InAir}}, e_{\text{FallingDown}}, s_{\text{TouchGround}})$$

should be consistent with:

$$\text{Meets}(t_{\text{FallingDown}}, t_{\text{TouchGround}})$$

That is, the interval over which the ball was falling down is immediately followed by the point when the ball touched the ground, but does not include the point itself. In other word, the ball was falling down immediately before the instant when it touched the group, but at the time point when the ball touched the ground, the ball was no longer falling down. Again, such a scenario is not allowed in Terenziani and Torasso’s Theorem 2.

5 Conclusions

Based on a simple point-based time model which allows expression of both absolute time values and relative temporal relations, we have presented in this paper a framework for representing flexible temporal causal relationships. The formalism presented here formally specifies the most general temporal constraint (GTC), ensuring the common-sense assertion that “the beginning of the effect cannot precede the beginning of the cause”. It is shown that the causal relationships characterized here are versatile enough to subsume those representatives in the literature. Ideally, any useful theory about action/event and change has to be able to handle the frame problem adequately. However, due to the length of this paper, we didn’t tackle such a problem. An interesting topic for further research is to extend this framework to include representing and reasoning about concurrent actions/events and their effects.

References

1. Allen J.: Maintaining Knowledge about Temporal Intervals, *Communication of ACM*. Vol. (26), (1983) 832-843.
2. Allen J.: Towards a General Theory of Action and Time, *Artificial Intelligence*, Vol. 23, (1984) 123-154.
3. Allen J. and Ferguson G.: Actions and Events in Interval Temporal Logic, *the Journal of Logic and Computation*, Vol. (4:5), (1994) 531-579.
4. Allen J. and Hayes P.: Moments and Points in an Interval-based Temporal-based Logic, *Computational Intelligence*, Vol. (5), (1989) 225-238.
5. Baral C.: Reasoning about actions: non-deterministic effects, constraints, and qualification. In *Proceedings of IJCAI’95*, (1995) 2017-2023.
6. Baral C. and Gelfond M.: Reasoning about Effects of Concurrent Actions. *Journal of Logic Programming*, vol. (31:1-3), (1997) 85-117.
7. Baral C, Son T. and Tuan L.: A transition function based characterization of actions with delayed and continuous effects. In *Proceedings of KR’02*, (2002) 291-302.
8. van Benthem J.: *The Logic of Time*, Kluwer Academic, Dordrech, (1983).
9. Galton A.: A Critical Examination of Allen’s Theory of Action and Time, *Artificial Intelligence*, Vol. (42), (1990) 159-188.
10. Gelfond M., Lifschitz V. and Rabinov A.: What are the Limitations of the Situation Calculus? In *Working Notes of AAAI Spring Symposium Series. Symposium: Logical Formalization of Commonsense Reasoning*, (1991) 59-69.
11. Kowalski R. and Sergot M.: A Logic-based Calculus of Events, *New Generation Computing*, Vol. (4), (1986) 67-95.
12. Lifschitz V.: Formal theories of action, In *Proceedings of the Tenth International Joint Conference on Artificial Intelligence*, (1987) 966-972.
13. Lin F. and Shoham Y.: Concurrent Actions in the Situation Calculus, in *Proceedings of AAAI-92*, (1992) 590-595.
14. Ma, J. and Hayes, P.: Primitive Intervals Vs Point-Based Intervals: Rivals Or Allies?”, *the Computer Journal*, Vol.49(1). (2006) 32-41
15. Ma J. and Knight B.: A General Temporal Theory, *The Computer Journal*, Vol. (37:2), (1994) 114-123.
16. Ma, J. and Knight B.: A Reified Temporal Logic, *The Computer Journal*, Vol. (39:9), (1996) 800-807.

17. McCarthy J.: Situations, actions and causal laws, Stanford Artificial Intelligence Project: Memo 2, (1963).
18. McCarthy J. and Hayes P. J.: Some philosophical problems from the standpoint of artificial intelligence, in *Machine Intelligence*, Vol. (4), Eds. Meltzer B. and Michie D., Edinburgh University Press, (1969) 463-502.
19. McDermott D.: A Temporal Logic for Reasoning about Processes and Plans, *Cognitive Science*, Vol. (6), (1982) 101-155.
20. Miller R. and Shanahan M.: Narratives in the Situation Calculus, the *Journal of Logic and Computation*, Vol. (4:5), (1994) 513-530.
21. Pinto J. and Reiter R.: Temporal Reasoning in Logic Programming: A Case for the Situation Calculus, In *Proceedings of 10th Int. Conf. on Logic Programming*, Budapest, Hungary, (1993) 21-24.
22. Pinto J. and Reiter R.: Reasoning about Time in the Situation Calculus, *Annals of Mathematics and Artificial Intelligence*, Vol. (14:2-4), (1995) 251-268.
23. Sandewall E.: Filter preferential entailment for the logic of action in almost continuous worlds. In *Proceedings of the 12th International Joint Conference on Artificial Intelligence*, (1989) 894-899.
24. Schubert L.: Monotonic Solution of the Frame Problem in the Situation Calculus: an Efficient Method for Worlds with Fully Specified Actions, in: H.E. Kyburg, R. Loui and G. Carlson, eds., *Knowledge Representation and Defeasible Reasoning* pp.23-67, Kluwer Academic Press, (1990).
25. Shanahan M.: A Circumscriptive Calculus of Events, *Artificial Intelligence*, Vol. (77), (1995) 29-384.
26. Shoham Y.: Temporal logics in AI: Semantical and Ontological Considerations, *Artificial Intelligence*, Vol (33), (1987) 89-104.
27. Shoham Y.: *Reasoning about Change: Time and Causation from the Standpoint of Artificial Intelligence*, MIT Press, (1988).
28. Terenziani P. and Torasso P.: Time, Action-Types, and Causation: an Integrated Analysis, *Computational Intelligence*, Vol. (11:3), (1995) 529-552.
29. Vila, L.: A survey on temporal Reasoning in Artificial Intelligence. *AI Communications*, Vol. (7), (1994) 4-28.